Scaling in Laminar Natural Convection in Laterally Heated Cavities — Is Turbulence Essential in the Classical Scaling of Heat Transfer?

Natural convection in a gravitational field is ubiquitous in nature and in many important technological systems and has been the subject of extensive studies in theory, experiments and numerical simulations (cf., reviews [1] and references therein). In addition to its importance in oceanography, geophysics, meteorology, astrophysics, energy systems and process technologies, Rayleigh-Benard convection (RBC), a special form of natural convection driven by a vertical temperature gradient, is also a classic system for the study of buoyancy-induced turbulence. One of the most dramatic discoveries from classical experiments on this system in the turbulent convection regime is the (effective) power law dependencies of heat transport on dimensionless buoyancy forcing with scaling exponents between 1/4 and 1/3 [2]. Significant effort, both numerically and experimentally, has been directed at the mechanisms and detailed scaling behavior of turbulent RBC.

Another configuration, natural convection driven by a horizontal temperature gradient, is just as important in practical applications but has received much less attention from the physics community. It is mostly selected as a validation problem to compare numerical algorithms designed for solving the Navier-Stokes or Boltzmann equations, or for turbulence modeling and computation. Theoretical, numerical and experimental work on this system has investigated flow patterns, temperature distributions, flow instabilities, etc., mostly focusing more on the transition to unsteady flow or the effects of aspect ratio on heat transfer at moderately high Ra.

We demonstrate that natural convection in these two configurations share some important characteristics. Although the flow regimes can be very different in that one is completely laminar whereas the other is governed, at least in the bulk, by strong turbulent fluctuations, the heat transfer scaling with forcing is very similar.

Natural convection is characterized by the Rayleigh number $Ra = g\alpha\Delta T d^3/\nu\kappa$ and the Prandtl number $Pr = \nu/\kappa$ with g the acceleration of gravity, α the thermal expansion coefficient, ΔT the applied temperature difference, d the distance along the temperature gradient, and ν and κ the kinematic viscosity and thermal diffusivity, respectively. The global response to buoyant forcing is measured by the thermal diffusion normalized heat transfer called the Nusselt number Nu, the mean (or max) flow velocity u and the associated inertia to dissipation ratio called Reynolds number, $Re = ud/\nu$. The last is for turbulent convection only.

In this study, we numerically investigate in detail power-law scaling of Nu on Ra in laminar convection subject to horizontal temperature gradient that is nearly identical to the classic scaling in turbulent RBC. Based on detailed examinations we find that the existence of a large-scale circulation (LSC) and the resultant boundary layers and interior temperature distribution is sufficient to produce the classic near 1/3 power-law scaling. We thus conjecture that (1) although turbulence produces rich non-universal flow dynamics it has little effect on the global heat transfer. (2) Similar near 1/3 power-law scaling is a universal characteristic for thermal convection and should exist in different flow regimes in all closed cavities with various temperature gradient arrangements. (3) Side-heated and tilted cavities may provide alternative routes to study the effects of turbulence on heat transfer in natural convection.

We perform simulations of the dimensionless Boussinesq equations in a square two-dimensional cavity ($\Gamma = 1$) with Pr =0.71. The boundary conditions are hot ($\Theta_h = 0.5$) at the left wall, cold ($\Theta_l = -0.5$) at the right wall, and adiabatic at the top and bottom boundaries. All four boundaries are non-slip. Initially, we set $\vec{v} = \Theta = 0$ everywhere within the cavity so that the flow is driven only by the buoyant force.

Two important properties for convective flow are Nu, which measures the enhancement of heat transfer by convection over conduction, and v_{max}, the maximum velocity amplitude in the field. The former is computed by $Nu = \int_0^1 (u\Theta - \frac{\partial\Theta}{\partial x})|_x dz$ where the first term is the contribution from heat convection and the second is from heat conduction. By definition $Nu = 1$ for conduction. We compute Nu at $x = 0.5$ although any value of x within the width from 0 to 1 gives the same Nu.

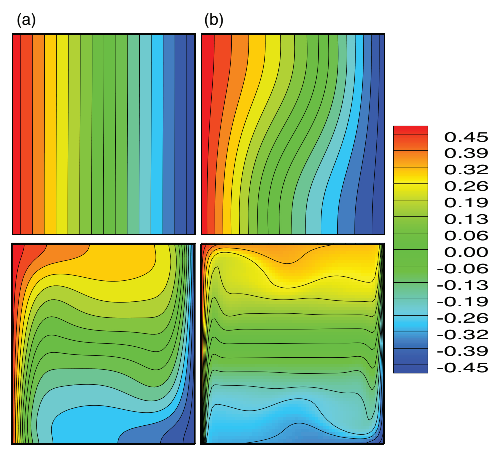

FIG. 1: Stationary temperature contours at Ra numbers (a) 10, (b) 10^3, (c) 10^5, (d) 10^7. Color coded temperature scale is shown on right.

Stationary temperature contours at different Ra are shown in Fig. 1. At low $Ra(= 10)$, the temperature gradient is distributed nearly uniformly over the whole field in the horizontal direction. Convection is weak and conduction dominates heat transfer. As Ra increases to about 10^3, buoyant forces becomes stronger and convection starts to play a role. The temperature distribution is deformed, and boundary layers begin to form along both sides. As Ra increases to 10^5, boundary layers are well developed and the fluid becomes thermally stratified. At this stage, heat transfer within the thin boundary layers is dominated by conduction and by convection outside. For $Ra \gtrsim 10^7$, the flow is further stratified and the boundary layers become very thin.

Quantitative measurements yield the dependence of Nu and v_{max} on Ra, see Figs. 2a,b. Two stages in the laminar flow regime are captured. At low Ra, Nu remains approximately unity indicating that conduction dominates. The growth of v_{max} is linear in Ra. In the second stage, both v_{max} and Nu exhibit power-law growth. The velocity magnitude scales as $v_{max} \sim 0.14Ra^{0.54}$, close to the experimental measurement [3] for turbulent RBC where the the scaling exponent for the velocity is 0.49. The dependence of Nu on Ra, $Nu \sim 0.13Ra^{0.31}$, shows excellent correspondence with turbulent RBC results with similar scaling exponent. This scaling exponent is consistent with previous numerical results [5] but is interpreted here in a new way.

The dependency of heat transfer Nu on cavity aspect-ratio Γ is computed in the range of $\Gamma = 0.5 - 20$ at constant Ra. Nu monotonically decreases from 5 at $\Gamma = 0.5$ to 2.4 at $\Gamma = 20$, in qualitative agreement with the classical experimental data [4].

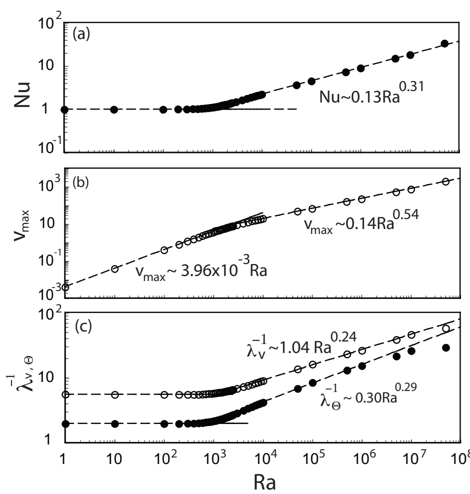

FIG. 2: Power-law scalings as a function of Ra. (a). Heat transfer Nu; (b). Maximum velocity magnitude v_{max}; (c) Mean inverse boundary layer thickness for velocity (dots) and temperature (circles).

As mentioned above, the power-law scaling ($Nu \sim Ra^{\beta}$) of large Ra number RBC has been well investigated. Experimental data reveal power law dependencies of $\beta = 0.25 \sim 0.33$, see Table 1 of reference [1]. Generally in turbulent RBC, mixing length theories predicts $Nu \sim Ra^{1/3}$ for $Pr \gtrsim 0.1$ [1] by supposing that the heat conduction is confined to the regions near the heated (or cooled) plates and that the two boundary layers don't communicate. From more general considerations, one gets a complicated diagram of regions with different power-law scalings for Nu between 1/4 and 1/2 [2] depending on the Ra and Pr numbers so that there may not be a pure power-law for experiments that cross over from one region to another [6]. In a recent *Nature* article [7], Niemela *et al* reported the experimental measurement of $Nu = 0.124Ra^{0.309}$ in the broadest range of Ra, from 10^6 to 10^{17}, using cryogenic helium gas near its critical point. This is remarkably close to the numerical result here ($0.13Ra^{0.31}$) over a wide range of Ra.

We further characterize the boundary layer thickness development in the laminar convection flow regime. We define the x-location of maximum vertical velocity magnitude (w) as the VBL thickness $\lambda_v(z)$ and the x-location of the intersept of the slope of temperature gradient at the hot wall with the corresponding stratified temperature line as the TBL thickness $\lambda_\Theta(z) = (\Theta_h - \Theta|_{(0.5,z)})/\frac{\partial \Theta}{\partial x}|_{(0,z)}$.

Figure 2c shows the mean inverse VBL and TBL thicknesses computed as $\lambda_v^{-1} = \sum_z (1/\lambda_v(z))$ and $\lambda_\Theta^{-1} = \sum_z (1/\lambda_\Theta(z))$ for different Ra. First, $\lambda_v < \lambda_\Theta$ for all Ra's, which is true for the $Pr < 1$ case. Second, according to the analysis above, after the fluid becomes stratified, heat transfer near the side boundaries is dominated by conduction. Therefore, the growth of Nu should be roughly proportional to λ_Θ^{-1}. Comparing $Nu \sim 0.13Ra^{0.31}$ in Fig. 2a with $\lambda_\Theta^{-1} \sim 0.30Ra^{0.29}$ in Fig. 2c we confirm this prediction ($\lambda_\Theta^{-1} \sim 2Nu$ noticing the total TBL thickness is $2\lambda_\Theta$). It is seen that Nu grows slightly faster than λ_Θ^{-1}. This is because that convection still makes a small contribution even in the boundary layers. Last and most importantly, the demonstration of boundary layer effects on the flow in different Ra ranges in turn helps in understanding the flow physics. For low Ra, the VBL and the TBL are not meaningful. The computed TBL is essentially half of the cavity width. After the flow starts to stratify, the VBL and the TBL form and begin to dominate heat transfer. The reduction of the boundary layer thickness follows a power law. As Ra reaches about 5×10^7, the flow becomes time dependent and perhaps turbulent, and the computation of the thickness of TBL and VBL can no longer utilize the same approach. Experimental measurement of thermal boundary layer thickness in turbulence gas convection driven by sidewall heating, defined as the position at which the temperature rms is maximum, shows the scaling with Ra to have an exponent of 0.29 in the range of Ra from 5×10^5 to 10^{11}, which is in agreement with the numerical results here. This is identical to that found in RBC with a slightly larger prefactor [9].

Figure 3 zooms in on the growth of Nu at very low $Ra(< 10^3)$. After a short rounded region where $Nu \sim Ra^2$, Nu becomes linear in Ra. These two scalings were predicted by Batchelor in 1954 [8]. We extract a threshold Rayleigh number Ra_t analogous to the critical Ra_c in RBC through the extrapolation of the linearity to the base value of $Nu = 1$ for pure conduction. This threshold Ra_t characterizes the onset of significant convection influence on the heat transfer. It is found that Ra_t is nearly independent of the cavity aspect-ratio.

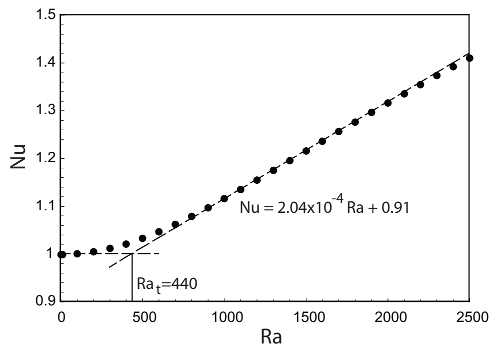

FIG. 3: Threshold Ra_t through extrapolation of the linear region to the base value of conduction, $Nu = 1$.

In what follows, we perform an analysis [10] analogous to RBC to interpret the near 1/3 power-law of Nu. For $Ra < Ra_t$, see Figs. 1 and 2, buoyancy induced flow is very weak and heat transfer is dominated by conduction. When Ra exceeds Ra_t, convection starts to dominate the heat transfer. Large scale shear flow forms as Ra reaches around 10^5. At this stage, the core region is stratified with large convective flow transporting heat, and there is effectively no temperature gradient horizontally. All significant horizontal temperature gradients are in two boundary layers with a total thickness $\lambda_t (= 2\lambda_\Theta)$. The conduction dominated boundary layer thickness may be determined by $Ra_t = (g\alpha\Delta T\lambda_t^3)/(\nu\kappa)$ whereas for the cavity $Ra = (g\alpha\Delta T L^3)/(\nu\kappa)$. Since the heat flux $j_h = \kappa\Delta T/\lambda_t$ and conduction flux $j_c = \kappa\Delta T/L$, by definition we have $Nu = j_h/j_c = L/\lambda_t = (Ra/Ra_t)^{1/3} = 0.13Ra^{1/3}$. This result slightly over predicts Nu obtained from the simulations. Following the same simple procedure for RBC, and not using the marginal stability argument for the boundary layer thickness, $Nu = (Ra/Ra_c)^{1/3} = 0.084Ra^{1/3}$. In addition, the natural existence of a LSC in the laterally heated cavities and the attainment of 0.31 power-law nearly identical to that in RBC suggests the modification of LSC to the 1/3 power-law may have some universal features.

This work systematically examines the heat transfer and flow properties in laminar natural convection in laterally heated cavities with numerical simulations and demonstrates the fundamental role of LSC in natural convection in closed cavities. The transition from conduction-dominated heat transfer to a convection-dominated regime, first proposed in the theoretical work of Batchelor, is clearly analyzed. The resulting threshold Ra_t, akin to the role of the critical Ra_c in RBC, is explored and used in the study of the transition to power scaling in Nu, velocity, and thermal and viscous boundary layers. Such scalings, all existing in laminar flow, are found to be nearly identical to those in turbulent RBC, prompting us to propose that LSC and the resultant boundary layers and stratified interior are sufficient to produce the classical near 1/3 (0.31) power-law scaling for heat transfer in all natural convection in closed cavities, and that the role of turbulence is not essential in this regard. This work establishes a new paradigm in the study of natural convection in closed cavities, and offers alternative routes to study the effects of LSC and turbulence.

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